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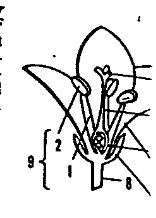
worth two shillings b : any of several similar coins issued in Commonwealth countries 3 : GULDEN 4 : FORINT
flo·rist \'flōr-ist, 'flȯr-, 'flär-\ *n* (1623) : one who sells or grows for sale flowers and ornamental plants — **flo·rist·ry** \-ə-strē\ *n*
flo·ris·tic \flō-'ris-tik\ *adj* (1898) : of or relating to flowers, a flora, or the phytogeographical study of plants and plant groups — **flo·ris·ti·cal·ly** \-ti-k(ə-)lē\ *adv*
flo·ru·it \'flōr-(y)ə-wət, 'flär-\ *n* [L, he flourished, fr. *florēre* to flourish] (1843) : a period of flourishing (as of a person or movement)
¹**floss** \'fläs, 'flȯs\ *n* [prob. modif. of F *floche* soft, weak (of silk fiber), fr. Gascon, fr. L *fluxus*, lit., loose, flowing, pp. of *fluere* to flow — more at FLUID] (1759) 1 a : soft thread of silk or mercerized cotton for embroidery b : DENTAL FLOSS 2 : fluffy fibrous material
²**floss** *vi* (1974) : to use dental floss on ~ *vi* : to use dental floss
flossy \'flä-sē, 'flȯ-\ *adj* **floss·i·er**; **-est** (1839) 1 : of, relating to, or having the characteristics of floss 2 : stylish or glamorous esp. at first impression ⟨~ new hotels⟩ — **floss·i·ly** \'flä-sə-lē\ *adv*
flo·ta \'flō-tə\ *n* [Sp] (1527) : a fleet of Spanish ships
flo·ta·tion \flō-'tā-shən\ *n* [²*float*] (1806) 1 : the act, process, or state of floating 2 : an act or instance of financing (as an issue of stock) 3 : the separation of the particles of a mass of pulverized ore according to their relative capacity for floating on a given liquid; *also* : any of various similar processes involving the relative capacity of materials for floating 4 : the ability (as of a tire or snowshoes) to stay on the surface of soft ground or snow
flo·til·la \flō-'ti-lə\ *n* [Sp, dim. of *flota* fleet, fr. OF *flote*, fr. ON *floti*; akin to OE *flota* ship, fleet — more at FLOAT] (1711) 1 : a fleet of ships or boats; *esp* : a navy organizational unit consisting of two or more squadrons of small warships 2 : an indefinite large number ⟨a ~ of changes⟩
flot·sam \'flät-səm\ *n* [AF *floteson*, fr. OF *floter* to float, of Gmc origin; akin to OE *flotian* to float, *flota* ship] (ca. 1607) 1 : floating wreckage of a ship or its cargo; *broadly* : floating debris 2 a : a floating population (as of emigrants or castaways) b : an accumulation of miscellaneous or unimportant stuff
¹**flounce** \'flaun(t)s\ *vi* **flounced**; **flounc·ing** [perh. of Scand origin; akin to Norw *flunsa* to hurry] (1542) 1 a : to move with exaggerated jerky or bouncy motions ⟨*flounced* about the room, jerking her shoulders, gesticulating —Agatha Christie⟩; *also* : to move so as to draw attention to oneself ⟨*flounced* into the lobby⟩ b : to go with sudden determination ⟨*flounced* out of the room in a huff⟩ 2 : FLOUNDER, STRUGGLE
²**flounce** *n* (1583) : an act or instance of flouncing — **flouncy** \'flaun(t)-sē\ *adj*
³**flounce** *vt* **flounced**; **flounc·ing** [alter. of earlier *frounce*, fr. ME *frouncen* to curl] (1711) : to trim with flounces
⁴**flounce** *n* (1713) : a strip of fabric attached by one edge; *also* : a wide ruffle — **flouncy** \'flaun(t)-sē\ *adj*
flounc·ing \'flaun(t)-siŋ\ *n* (1865) : material used for flounces
¹**floun·der** \'flaun-dər\ *n, pl* **flounder** *or* **flounders** [ME, fr. AF *floundre*, of Scand origin; akin to ON *flythra* flounder] (15c) : FLATFISH; *esp* : a fish of either of two families (Pleuronectidae and Bothidae) that include important marine food fishes
²**flounder** *vi* **floun·dered**; **floun·der·ing** \-d(ə-)riŋ\ [prob. alter. of *founder*] (1592) 1 : to struggle to move or obtain footing : thrash about wildly 2 : to proceed or act clumsily or ineffectually
¹**flour** \'flaur\ *n* [ME — more at FLOWER] (13c) 1 : finely ground meal of wheat usu. largely freed from bran; *also* : a similar meal of another material (as a cereal grain, an edible seed, or dried processed fish) 2 : a fine soft powder — **flour·less** *adj* — **floury** \-ē\ *adj*
²**flour** *vt* (ca. 1657) : to coat with or as if with flour ~ *vi* : to break up into particles
¹**flour·ish** \'flər-ish, 'flə-rish\ *vb* [ME *florisshen*, fr. MF *floriss-*, stem of *florir*, fr. (assumed) VL *florire*, alter. of L *florēre*, fr. *flor-*, *flos* flower] *vi* (14c) 1 : to grow luxuriantly : THRIVE 2 a : to achieve success : PROSPER b : to be in a state of activity or production ⟨~*ed* around 1850⟩ c : to reach a height of development or influence 3 : to make bold and sweeping gestures ~ *vt* : to wield with dramatic gestures : BRANDISH **syn** see SWING — **flour·ish·er** *n* — **flour·ish·ing·ly** \-i-shiŋ-lē\ *adv*
²**flourish** *n* (1597) 1 a : a period of thriving b : a luxuriant growth or profusion ⟨a ~ of white hair⟩ ⟨a springtime ~ of color⟩ 2 a : a florid bit of speech or writing ⟨rhetorical ~*es*⟩ b : an ornamental stroke in writing or printing c : a decorative or finishing detail ⟨a house with clever little ~*es*⟩ 3 : FANFARE 4 : an act or instance of brandishing or waving 5 : showiness in the doing of something ⟨opened the door with a ~⟩ 6 : a sudden burst (as of activity) ⟨the week ends with a ~ of tests⟩
¹**flout** \'flaut\ *vb* [prob. fr. ME *flouten* to play the flute, fr. *floute* flute] *vt* (1551) : to treat with contemptuous disregard : SCORN ⟨~*ing* the rules⟩ ~ *vi* : to indulge in scornful behavior **syn** see SCOFF **usage** see FLAUNT — **flout·er** *n*
²**flout** *n* (ca. 1570) : JEER
¹**flow** \'flō\ *vb* [ME, fr. OE *flōwan*; akin to OHG *flouwen* to rinse, wash, L *pluere* to rain, Gk *plein* to sail, float] *vi* (bef. 12c) 1 a (1) : to issue or move in a stream (2) : CIRCULATE b : to move with a continual change of place among the constituent particles ⟨molasses ~*s* slowly⟩ 2 : RISE ⟨the tide ebbs and ~*s*⟩ 3 : ABOUND 4 a : to proceed smoothly and readily ⟨conversation ~*ed* easily⟩ b : to have a smooth continuity 5 : to hang loose and billowing 6 : to derive from a source : COME ⟨the wealth that ~*s* from trade⟩ 7 : to deform under stress without cracking or rupturing — used esp. of minerals and rocks 8 : MENSTRUATE ~ *vt* 1 : to cause to flow 2 : to discharge in a flow **syn** see SPRING — **flow·ing·ly** \-iŋ-lē\ *adv*
²**flow** *n* (15c) 1 : an act of flowing 2 : FLOOD 1a, 2 3 a : a smooth uninterrupted movement or progress ⟨a ~ of information⟩ b : STREAM; *also* : a mass of material which has flowed when molten ⟨an old lava ~⟩ c : the direction of movement or development ⟨go with the ~⟩ 4 : the quantity that flows in a certain time 5 : MENSTRUATION 6 a : the motion characteristic of fluids b : a continuous transfer of energy
flow·age \'flō-ij\ *n* (1830) 1 a : an overflowing onto adjacent land b : a body of water formed by overflowing or damming c : floodwater esp. of a stream 2 : gradual deformation of a body of (as rock) by intermolecular shear
flow·chart \-,chärt\ *n* (1920) : a diagram that shows step gression through a procedure or system esp. using connect a set of conventional symbols — **flow·chart·ing** \-,chär-ti
flow cy·tom·e·try \-sī-'tä-mə-trē\ *n* (1978) : a technique ing and sorting cells and their components (as DNA) by a fluorescent dye and detecting the fluorescence usu. by illumination
flow diagram *n* (1943) : FLOWCHART
¹**flow·er** \'flau(-ə)r\ *n* [ME *flour* flower, best of anything, flour, fr. OF *flor*, *flour*, fr. L *flor-*, *flos* — more at BLOW] (13c) 1 a : BLOSSOM, INFLORESCENCE b : a shoot of the sporophyte of a higher plant that is modified for reproduction and consists of a shortened axis bearing modified leaves; *esp* : one of a seed plant differentiated into a calyx, corolla, stamens, and carpels c : a plant cultivated for its blossoms 2 a : the best part or example ⟨the ~ of our youth⟩ b : the finest most vigorous period c : a state of blooming or flourishing ⟨in full ~⟩ 3 *pl* : a finely divided powder produced esp. by condensation or sublimation ⟨~*s* of sulfur⟩ — **flow·ered** \'flau(-ə)rd\ *adj* — **flow·er·ful** \'flau(-ə)r-fəl\ *adj* — **flow·er·less** \-ləs\ *adj* — **flow·er·like** \-,līk\ *adj*



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filament, 2 anther,
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²**flower** *vi* (13c) 1 a : DEVELOP ⟨~*ed* into young woma : FLOURISH 2 : to produce flowers : BLOSSOM ~ *vt* 1 : bear flowers 2 : to decorate with flowers or floral designs — **flow·er·er** \'flau(-ə)r-ər\ *n*
flow·er·age \'flau(-ə)r-ij\ *n* (1840) : a flowering process, sta dition
flower bud *n* (1828) : a plant bud that produces only a flowe
flower bug *n* (ca. 1889) : any of various small mostly black predaceous bugs (family Anthocoridae) that frequent flowe on pest insects (as aphids and thrips)
flower child *n* (1967) : a hippie who advocates love, beauty,
flow·er·et *also* **flow·er·ette** \'flau(-ə)r-ət\ *n* (15c) : FLORET
flower girl *n* (1925) : a little girl who carries flowers at a wed
flower head *n* (1845) : a capitulum (as of a composite) hav flowers so arranged that the whole inflorescence looks lik flower
flowering dogwood *n* (1843) : a common spring-floweri bracted dogwood (*Cornus florida*)
flowering plant *n* (1745) : ANGIOSPERM
flower people *n pl* (1967) : FLOWER CHILDREN
flow·er·pot \'flau(-ə)r-,pät\ *n* (1598) : a pot in which to grow
flow·ery \'flau(-ə)r-ē\ *adj* (14c) 1 : of, relating to, or resemb ers 2 : marked by or given to rhetorical elegance — **flo** \'flau(-ə)r-ə-lē\ *adv* — **flow·er·i·ness** *n*
flow·me·ter \'flō-,mē-tər\ *n* (1915) : an instrument for meas or more properties (as velocity or pressure) of a flow (as of a l pipe)
¹**flown** \'flōn\ *past part of* FLY
²**flown** *adj* [archaic pp. of ¹*flow*] (1626) : filled to excess
flow sheet *n* (1912) : FLOWCHART
flow·stone \'flō-,stōn\ *n* (1925) : calcite deposited by a thin flowing water usu. along the walls or floor of a cave
flu \'flü\ *n* [by shortening] (1839) 1 : INFLUENZA 2 : any virus diseases marked esp. by respiratory symptoms
¹**flub** \'fləb\ *vb* **flubbed**; **flub·bing** [origin unknown] *vt* (1 make a mess of : BOTCH ⟨*flubbed* my lines⟩ ~ *vi* : BLUNDER
²**flub** *n* (1948) : an act or instance of flubbing
flub·dub \'fləb-,dəb\ *n* [origin unknown] (1888) : BUNKUM, DASH
fluc·tu·ant \'flək-chə-wənt\ *adj* (1560) 1 : moving in waves ABLE, UNSTABLE 3 : being movable and compressible ⟨a ~ absc
fluc·tu·ate \'flək-chə-,wāt\ *vb* **-at·ed**; **-at·ing** [L *fluctuatus*, *fluctuare*, fr. *fluctus* flow, wave, fr. *fluere* — more at FLUID] *vi* (1 : to shift back and forth uncertainly 2 : to ebb and flow in *vt* : to cause to fluctuate **syn** see SWING — **fluc·tu·a·tion** \ 'wā-shən\ *n* — **fluc·tu·a·tion·al** \-'wā-shnəl, -shə-nᵊl\ *adj*
flue \'flü\ *n* [origin unknown] (1582) : an enclosed passage directing a current: as a : a channel in a chimney for convey and smoke to the outer air b : a pipe for conveying flame gases around or through water in a steam boiler c : an air leading to the lip of a wind instrument d : FLUE PIPE
flue–cured \-,kyürd\ *adj* (1905) : cured with heat transmitted a flue without exposure to smoke or fumes ⟨~ tobacco⟩
flu·en·cy \'flü-ən(t)-sē\ *n* (1636) : the quality or state of being
flu·ent \'flü-ənt\ *adj* [L *fluent-*, *fluens*, prp. of *fluere*] (1599) 1 pable of flowing : FLUID b : capable of moving with ease ⟨the ~ body of a dancer⟩ 2 a : ready or facile in speech ⟨~ ish⟩ b : effortlessly smooth and rapid : POLISHED ⟨a ~ perfor — **flu·ent·ly** *adv*
flue pipe *n* (1852) : an organ pipe whose tone is produced by current striking the lip and causing the air within to vibrate pare REED PIPE
flue stop *n* (1855) : an organ stop made up of flue pipes
¹**fluff** \'fləf\ *n* [perh. blend of *flue* (fluff) and *puff*] (1790) 1 : DOWN 2 : something fluffy 3 : something inconsequential DER; *esp* : an actor's lapse of memory
²**fluff** *vi* (1875) 1 : to become fluffy 2 : to make a mistake forget or bungle one's lines in a play ~ *vt* 1 : to make fluffy : to spoil by a mistake : BOTCH b : to deliver badly or forget lines) in a play
fluffy \'flə-fē\ *adj* **fluff·i·er**; **-est** (ca. 1825) 1 a : covered resembling fluff b : being light and soft or airy ⟨a ~ omelet

: lacking in meaning or substance : SUPERFICIAL 2c — fluff·i·ly \'flə-fə-lē\ adv — fluff·i·ness \'flə-fē-nəs\ n

flü·gel·horn or flue·gel·horn \'flü-gəl-,hȯrn, 'flīē-\ n [G, fr. Flügel wing, flank + Horn horn; fr. its use to signal the flanking drivers in a battue] (1854) : a valved brass instrument resembling a cornet but having a larger bore — flü·gel·horn·ist \-,hȯr-nist\ n

¹flu·id \'flü-əd\ adj [F or L; F fluide, fr. L fluidus, fr. fluere to flow; akin to Gk phlyzein to boil over] (1603) 1 a : having particles that easily move and change their relative position without a separation of the mass and that easily yield to pressure : capable of flowing b : subject to change or movement ⟨boundaries became ~⟩ 2 : characterized by or employing a smooth easy style ⟨the ballerina's ~ movements⟩ 3 a : available for a different use b : LIQUID 4 ⟨~ assets⟩ — flu·id·ly adv — flu·id·ness n

²fluid n (1661) : a substance (as a liquid or gas) tending to flow or conform to the outline of its container — flu·id·al \'flü-ə-dᵊl\ adj — flu·id·al·ly \-dᵊl-ē\ adv

fluid dram or flu·i·dram \,flü-ə(d)-'dram\ n (ca. 1860) : a unit of liquid capacity equal to 1/8 fluid ounce — see WEIGHT table

flu·id·ex·tract \,flü-ə-'dek-,strakt\ n (1851) : an alcohol preparation of a vegetable drug containing the active constituents of one gram of the dry drug in each milliliter

flu·id·ic \flü-'i-dik\ adj (1960) : of, relating to, or being a device (as an amplifier or control) that depends for operation on the pressures and flows of a fluid in precisely shaped channels — fluidic n — flu·id·ics \-diks\ n pl but sing in constr

flu·id·i·ty \flü-'i-də-tē\ n (1603) 1 : the quality or state of being fluid 2 : the physical property of a substance that enables it to flow

flu·id·ize \'flü-ə-,dīz\ vt -ized; -iz·ing (ca. 1855) 1 : to cause to flow like a fluid 2 : to suspend (as solid particles) in a rapidly moving stream of gas or vapor to induce flowing motion of the whole — flu·id·iza·tion \,flü-ə-də-'zā-shən\ n — flu·id·iz·er \'flü-ə-,dī-zər\ n

fluidized bed n (1949) : a bed of small solid particles (as in a coal burning furnace) suspended and kept in motion by an upward flow of a fluid (as a gas) — called also fluid bed

fluid mechanics n pl but sing or pl in constr (1937) : a branch of mechanics dealing with the properties of liquids and gases

fluid ounce n (ca. 1860) 1 : a U.S. unit of liquid capacity equal to 1/16 pint — see WEIGHT table 2 : a British unit of liquid capacity equal to 1/20 pint — see WEIGHT table

¹fluke \'flük\ n [ME, fr. OE flōc; akin to OE flōh chip, OHG flah smooth, Gk plax flat surface, and prob. to OE flōr floor — more at FLOOR] (bef. 12c) 1 : FLATFISH 2 : a flattened digenetic trematode worm; broadly : TREMATODE — compare LIVER FLUKE

²fluke n [perh. fr. ¹fluke] (1561) 1 : the part of an anchor that fastens in the ground — see ANCHOR illustration 2 : one of the lobes of a whale's tail

³fluke n [origin unknown] (1857) 1 : an accidentally successful stroke at billiards or pool 2 : a stroke of luck ⟨the discovery was a ~⟩

fluky also fluk·ey \'flü-kē\ adj fluk·i·er; -est (1867) 1 : happening by or depending on chance 2 : being unsteady or uncertain — used esp. of wind

flume \'flüm\ n [prob. fr. ME flum river, fr. OF, fr. L flumen, fr. fluere — more at FLUID] (1748) 1 : an inclined channel for conveying water (as for power) 2 : a ravine or gorge with a stream running through it

flum·mery \'flə-mə-rē, 'flə-mə-\ n, pl -mer·ies [W llymru] (1623) 1 a : a soft jelly or porridge made with flour or meal b : any of several sweet desserts 2 : MUMMERY, MUMBO JUMBO

flum·mox \'flə-məks, -miks\ vt [origin unknown] (1837) : CONFUSE

¹flump \'fləmp\ [imit.] vi (1816) : to move or fall suddenly and heavily ⟨~ed down into the chair⟩ ~ vt : to place or drop with a flump

²flump n (1832) : a dull heavy sound (as of a fall)

flung past and past part of FLING

¹flunk \'fləŋk\ vb [perh. blend of flinch and funk] vi (1823) : to fail esp. in an examination or course ~ vt 1 : to give a failing grade to 2 : to get a failing grade in — flunk·er n

²flunk n (1846) : an act or instance of flunking

flunk out vi (1920) : to be dismissed from a school or college for failure ~ vt : to dismiss from a school or college for failure

flun·ky or flun·key \'fləŋ-kē\ n, pl flunkies or flunkeys [Sc, of unknown origin] (ca. 1782) 1 a : a liveried servant b : one performing menial or miscellaneous duties 2 : YES-MAN

flu·o·cin·o·lone ac·e·to·nide \,flü-ə-'si-nᵊl-,ōn-,a-sə-'tō-,nīd\ n [fluor- + -cinolone, prob. alter. of -nisolone (as in prednisolone)] (1963) : a glucocorticoid steroid $C_{24}H_{30}F_2O_6$ used esp. as an anti-inflammatory agent in the treatment of skin diseases

flu·or \'flü-,ȯr, 'flü-ər\ n [NL, mineral belonging to a group used as fluxes and including fluorite, fr. L, flow, fr. fluere — more at FLUID] (1661) : FLUORITE

fluor- or fluoro- comb form [F] 1 : fluorine ⟨fluoride⟩ 2 also fluori- : fluorescence ⟨fluoroscope⟩ ⟨fluorimeter⟩

flu·o·resce \flü-'res, flō-, flȯ-\ vi -resced; -resc·ing [back-formation fr. fluorescence] (1874) : to produce, undergo, or exhibit fluorescence — flu·o·resc·er n

flu·o·res·ce·in \-'re-sē-ən\ n (1876) : a yellow or red crystalline dye $C_{20}H_{12}O_5$ with a bright yellow-green fluorescence in alkaline solution

flu·o·res·cence \-'re-sᵊn(t)s\ n [fluorspar + opalescence] (1852) : luminescence that is caused by the absorption of radiation at one wavelength followed by nearly immediate reradiation usu. at a different wavelength and that ceases almost immediately when the incident radiation stops; also : the radiation emitted — compare PHOSPHORESCENCE

flu·o·res·cent \-sᵊnt\ adj (1853) 1 : having or relating to fluorescence 2 : bright and glowing as a result of fluorescence ⟨~ inks⟩; broadly : very bright in color — fluorescent n

fluorescent lamp n (1896) : a tubular electric lamp having a coating of fluorescent material on its inner surface and containing mercury vapor whose bombardment by electrons from the cathode provides ultraviolet light which causes the material to emit visible light

flu·o·ri·date \'flür-ə-,dāt, 'flȯr-, 'flōr-\ vt -dat·ed; -dat·ing (1949) : to add a fluoride to (as drinking water) to reduce tooth decay — fluo·ri·da·tion \,flür-ə-'dā-shən, ,flȯr-, ,flōr-\ n

fluo·ride \'flȯr-,īd, 'flür-\ n, often attrib (1826) 1 : a compound of fluorine 2 : the monovalent anion of flourine

fluo·ri·nate \'flȯr-ə-,nāt, 'flōr-, 'flür-\ vt -nat·ed; -nat·ing (ca. 1929) : to treat or cause to combine with fluorine or a compound of fluorine — fluo·ri·na·tion \,flȯr-ə-'nā-shən, ,flōr-, ,flür-\ n

fluo·rine \'flür-,ēn, 'flȯr-, 'flōr-\ n [F, fr. NL fluor] (1813) : a nonmetallic halogen element that is isolated as a pale yellowish flammable irritating toxic diatomic gas — see ELEMENT table

fluo·rite \'flür-,īt, 'flȯr-, 'flōr-\ n [It, fr. NL fluor] (1868) : a transparent or translucent mineral of different colors that consists of the fluoride of calcium and is used as a flux and in the making of opalescent and opaque glasses

fluo·ro·car·bon \,flür-ō-'kär-bən, ,flȯr-, ,flōr-\ n (1937) : any of various chemically inert compounds containing carbon and fluorine used chiefly as lubricants, refrigerants, nonstick coatings, and formerly aerosol propellants and in making resins and plastics; also : CHLOROFLUOROCARBON

fluo·ro·chrome \'flür-ə-,krōm, ,flȯr-, ,flōr-\ n (1943) : any of various fluorescent substances used in biological staining to produce fluorescence in a specimen

fluo·rog·ra·phy \flü-'rä-grə-fē, flȯ-, flō-\ n (1941) : PHOTOFLUOROGRAPHY — flu·o·ro·graph·ic \,flür-ə-'gra-fik, ,flȯr-, flōr-\ adj

fluo·rom·e·ter \flü-'rä-mə-tər, flȯ-, flō-\ or fluo·rim·e·ter \-'ri-\ n (1897) : an instrument for measuring fluorescence and related phenomena (as intensity of radiation) — fluo·ro·met·ric or fluo·ri·met·ric \,flür-ə-'me-trik, ,flȯr-, ,flōr-\ adj — fluo·rom·e·try \flü-'rä-mə-trē, flȯ-, flō-\ or fluo·rim·e·try \-'ri-mə-trē\ n

¹fluo·ro·scope \'flür-ə-,skōp, 'flȯr-, 'flōr-\ n [ISV] (1896) : an instrument used for observing the internal structure of an opaque object (as the living body) by means of X rays — fluo·ro·scop·ic \,flür-ə-'skä-pik, ,flȯr-, ,flōr-\ adj — fluo·ro·scop·i·cal·ly \-pi-k(ə-)lē\ adv — fluo·ros·co·pist \flü-'räs-kə-pist, flȯ-, flō-\ n — fluo·ros·co·py \-pē\ n

²fluoroscope vt -scoped; -scop·ing (1898) : to examine by fluoroscopy

fluo·ro·sis \flü-'rō-səs, flȯ-\ n [NL] (1927) : an abnormal condition (as mottling of the teeth) caused by fluorine or its compounds — fluo·rot·ic \-'rä-tik\ adj

fluo·ro·ura·cil \,flür-ō-'yür-ə-,sil, -,səl, ,flȯr-, ,flōr-\ n [fluor- + uracil] (ca. 1958) : a fluorine-containing pyrimidine base $C_4H_3FN_2O_2$ used to treat some kinds of cancer

fluor·spar \'flür-,spär, 'flȯr-, 'flōr-\ n (1794) : FLUORITE

flu·phen·azine \flü-'fe-nə-,zēn\ n [fluor- + phenazine] (ca. 1960) : a tranquilizing compound $C_{22}H_{26}F_3N_3OS$ used esp. combined as a salt

¹flur·ry \'flər-ē, 'flə-rē\ n, pl flurries [prob. fr. flurr (to throw scatteringly)] (1686) 1 a : a gust of wind b : a brief light snowfall 2 a : a brief period of commotion or excitement b : a sudden occurrence of many things at once : BARRAGE 2 ⟨a ~ of insults⟩ 3 : a brief advance or decline in prices : a short-lived outburst of trading activity

²flurry vb flur·ried; flur·ry·ing vt (1757) : to cause to become agitated and confused ~ vi : to move in an agitated or confused manner

¹flush \'fləsh\ vb [ME flusshen] vi (13c) : to take wing suddenly ~ vt 1 : to cause (a bird) to flush 2 : to expose or chase from a place of concealment ⟨~ed the boys from their hiding place⟩

²flush n [MF flus, fluz, fr. L fluxus flow, flux] (ca. 1529) 1 : a hand of playing cards all of the same suit; specif : a poker hand containing five cards of the same suit but not in sequence — see POKER illustration 2 : a series of three or more slalom gates set vertically on a slope

³flush n [perh. modif. of L fluxus] (1529) 1 : a sudden flow (as of water); also : a rinsing or cleansing with or as if with a flush of water 2 a : a sudden increase or expansion; esp : sudden and usu. abundant new plant growth b : a surge of emotion ⟨felt a ~ of anger at the insult⟩ 3 a : a tinge of red : BLUSH b : a fresh and vigorous state ⟨in the first ~ of womanhood⟩ 4 : a transitory sensation of extreme heat — compare HOT FLASH

⁴flush vi (1548) 1 : to flow and spread suddenly and freely 2 a : to glow brightly b : BLUSH 3 : to produce new growth ⟨the plants ~ twice during the year⟩ ~ vt 1 a : to cause to flow b : to pour liquid over or through; esp : to cleanse or wash out with or as if with a rush of liquid ⟨~ the toilet⟩ ⟨~ the lungs with air⟩ 2 : INFLAME, EXCITE — usu. used passively ⟨~ed with pride⟩ 3 : to cause to blush

⁵flush adj (1594) 1 a : of a ruddy healthy color b : full of life and vigor : LUSTY 2 a : filled to overflowing b : AFFLUENT 3 : readily available : ABUNDANT 4 a : having or forming a continuous plane or unbroken surface ⟨~ paneling⟩ b : directly abutting or immediately adjacent: as (1) : set even with an edge of a type page or column : having no indention (2) : arranged edge to edge so as to fit snugly — flush·ness n

⁶flush adv (1700) 1 : in a flush manner 2 : SQUARELY ⟨hit him ~ on the chin⟩

⁷flush vt (ca. 1842) : to make flush ⟨~ the headings on a page⟩

flush·able \'flə-shə-bəl\ adj (1973) : suitable for disposal by flushing down a toilet

¹flus·ter \'fləs-tər\ vb flus·tered; flus·ter·ing \-t(ə-)riŋ\ [prob. of Scand origin; akin to Icel flaustur hurry] vt (1604) 1 : to make tipsy 2 : to put into a state of agitated confusion : UPSET ~ vi : to move or behave in an agitated or confused manner syn see DISCOMPOSE — flus·tered·ly adv

²fluster n (1728) : a state of agitated confusion

¹flute \'flüt\ n [ME floute, fr. MF fleute, fr. OF flaüte, prob. fr. OProv flaut] (14c) 1 a : RECORDER 3 b : a keyed woodwind instrument consisting of a cylindrical tube which is stopped at one end and which has a side hole over which air is blown to produce the tone and having a range from middle C upward for three octaves 2 : something

flute 1b

STATISTICAL MECHANICS

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CHAPTER 18

KINETIC THEORY OF GASES AND THE BOLTZMANN EQUATION

In Section 7–2 we introduced the concept of phase space and distribution functions in phase space. We also derived the Liouville equation, which is the equation of motion that the phase space distribution function must satisfy. Since we were interested only in equilibrium statistical mechanics at that time, we did not consider the Liouville equation in any detail. In this chapter we shall review the concept of phase space and derive the Liouville equation again. We shall then introduce reduced distribution functions and derive the Bogoliubov, Born, Green, Kirkwood, Yvon (BBGKY) hierarchy. This hierarchy is the nonequilibrium generalization of the Kirkwood integral equation hierarchy for the fluid distribution functions, $g^{(n)}(\mathbf{r}_1, \ldots, \mathbf{r}_n)$, of Chapter 13. Nobody has yet devised a successful way to uncouple the BBGKY hierarchy, and so in Section 18–4 we shall derive a physical, yet approximate, equation for the distribution function for gases. This equation, called the Boltzmann equation, is the central equation of the rigorous kinetic theory of gases. In Section 18–5, we shall derive some of the general consequences of the Boltzmann equation that can be determined without actually solving it completely. We shall discuss its solution in Chapter 19. The standard reference for most of this chapter is Hirschfelder, Curtiss, and Bird. Mazo (in "Additional Reading") also discusses these topics well.

18–1 PHASE SPACE AND THE LIOUVILLE EQUATION

Consider a system of N point particles. The classical dynamical state of this system is specified by the $3N$ momentum components $p_1, p_2, \ldots, p_{3N}$ and the $3N$ spatial coordinates $q_1, \ldots, q_{3N}$. We can construct a $6N$-dimensional space whose coordinates are $q_1, q_2, \ldots, p_1, \ldots, p_{3N}$. One point in this *phase space* completely specifies the microscopic dynamical state of our N-particle system. As the system evolves in time, this *phase point* moves through phase space in a manner completely dictated by the equations of motion of the system. Actually, one never knows (nor really cares to know) the $6N$ coordinates of a macroscopic system. Rather, one knows just a few

macroscopic mechanical properties of the system, such as the energy, volume, velocity, etc. Clearly there are a great number of points in phase space that are compatible with the few variables that we know about the system. The set of all such phase points constitutes an *ensemble of systems*. The number of systems in an ensemble approaches infinity, and so the set of phase points that could possibly represent our system becomes quite dense. This allows us to define a *density of phase points* or *distribution function* as the fraction of phase points contained in the volume $dq_1\, dq_2 \cdots dp_{3N}$. We shall denote the phase space distribution function by $f_N(q_1, q_2, \ldots, p_{3N}, t)$, or more conveniently by $f_N(p, q, t)$. We shall often use this abbreviated notation. Similarly, we shall often denote $dq_1, dq_2 \cdots dp_{3N}$ by $dp\, dq$. The density $f_N(p, q, t)$ is normalized such that

$$\int f_N(p, q, t)\, dp\, dq = 1$$

Since each phase point moves in time according to the equations of motion of the system it describes, f_N itself must obey some sort of equation of motion. The equation that $f_N(p, q, t)$ satisfies can be readily determined by using the methods of the previous chapter, particularly, the argument associated with Eqs. (17–1) to (17–5). The number of phase points within some arbitrary volume v is

$$n = \mathcal{N} \int_v f_N(p, q, t)\, dp\, dq$$

where we are using the condensed notation of letting p and q denote all the spatial coordinates and momenta necessary to specify a system in the ensemble. The rate of change of the number of phase points within v is

$$\frac{dn}{dt} = \mathcal{N} \int_v \frac{\partial f_N}{\partial t}\, dp\, dq \tag{18–1}$$

Since phase points are neither created nor destroyed, the rate of change of n must be given by the rate at which phase points flow through the surface enclosing v. The rate of flow of phase points is $\mathcal{N} f_N \mathbf{u}$, where $\mathbf{u}$ is not just the $3N$-dimensional vector $(\dot{q}_1, \dot{q}_2, \ldots, \dot{q}_{3N})$, but the $6N$-dimensional vector $(\dot{q}_1, \ldots, \dot{p}_1, \ldots, \dot{p}_{3N})$ since the spatial coordinates and momenta play an equivalent role in phase space. We integrate this flow over the surface to get

$$\frac{dn}{dt} = -\mathcal{N} \int_S f_N \mathbf{u} \cdot d\mathbf{S}$$

The negative sign here indicates that an outflow of phase points yields a negative value for dn/dt since $\mathbf{u} \cdot d\mathbf{S}$ is positive if $\mathbf{u}$ is directed outward from v and negative if $\mathbf{u}$ is directed inward.

The surface integral can be transformed to a volume integral by using Gauss' theorem to get

$$\frac{dn}{dt} = -\mathcal{N} \int_v \nabla \cdot (f_N \mathbf{u})\, dp\, dq \tag{18–2}$$

If we subtract Eq. (18–1) from Eq. (18–2) and realize that this equation is valid for any choice of v, we have the equation for the conservation of phase points

$$\frac{\partial f_N}{\partial t} + \nabla \cdot (f_N \mathbf{u}) = 0 \tag{18–3}$$

in which it should be clear that since we are dealing with phase space

$$\mathbf{u} = (\dot{q}_1, \ldots, \dot{q}_{3N}, \dot{p}_1, \ldots, \dot{p}_{3N})$$

and

$$\begin{aligned}\nabla \cdot f_N \mathbf{u} &= \sum_{j=1}^{3N} \frac{\partial}{\partial q_j}(f_N \dot{q}_j) + \sum_{j=1}^{3N} \frac{\partial}{\partial p_j}(f_N \dot{p}_j) \\ &= \sum_{j=1}^{3N} \left\{\frac{\partial f_N}{\partial q_j}\dot{q}_j + \frac{\partial f_N}{\partial p_j}\dot{p}_j\right\} + \sum_{j=1}^{3N}\left\{\frac{\partial \dot{q}_j}{\partial q_j} + \frac{\partial \dot{p}_j}{\partial p_j}\right\} f_N\end{aligned}$$

But Eq. (7–27) shows that the summand of the second summation here is zero, and so Eq. (18–3) becomes

$$\frac{\partial f_N}{\partial t} + \sum_{j=1}^{3N} \frac{\partial f_N}{\partial q_j}\dot{q}_j + \sum_{j=1}^{3N} \frac{\partial f_N}{\partial p_j}\dot{p}_j = 0 \tag{18–4}$$

Using Hamilton's equations of motion,

$$\dot{p}_i = -\frac{\partial H}{\partial q_i} \quad \text{and} \quad \dot{q}_i = \frac{\partial H}{\partial p_i}$$

Eq. (18–4) can be written

$$\frac{\partial f_N}{\partial t} + \sum_{j=1}^{3N}\left(\frac{\partial H}{\partial p_j}\frac{\partial f_N}{\partial q_j} - \frac{\partial H}{\partial q_j}\frac{\partial f_N}{\partial p_j}\right) = 0 \tag{18–5}$$

The summation here is called a Poisson bracket and is commonly denoted by $\{H, f_N\}$; so Eq. (18–5) is often written as

$$\frac{\partial f_N}{\partial t} + \{H, f_N\} = 0 \tag{18–6}$$

This is the Liouville equation, the most fundamental equation of statistical mechanics. In fact, it can be shown that the Liouville equation is equivalent to the $6N$ Hamilton equations of motion of the N-body system.*

In Cartesian coordinates, the Liouville equation reads

$$\frac{\partial f_N}{\partial t} + \sum_{j=1}^{N} \frac{\mathbf{p}_j}{m_j} \cdot \nabla_{\mathbf{r}_j} f_N + \sum_{j=1}^{N} \mathbf{F}_j \cdot \nabla_{\mathbf{p}_j} f_N = 0 \tag{18–6′}$$

In this equation $\nabla_{\mathbf{r}_j}$ denotes the gradient with respect to the spatial variables in f_N; $\nabla_{\mathbf{p}_j}$ denotes the gradient with respect to the momentum variables in f_N; and $\mathbf{F}_j$ is the total force on the jth particle.

One often sees the Liouville equation written as

$$i\frac{\partial f_N}{\partial t} = Lf_N \tag{18–7}$$

where L is the Liouville operator,

$$L = -i\left(\sum_{j=1}^{N} \frac{\mathbf{p}_j}{m_j} \cdot \nabla_{\mathbf{r}_j} + \sum_{j=1}^{N} \mathbf{F}_j \cdot \nabla_{\mathbf{p}_j}\right) \tag{18–8}$$

* Mazo, "Additional Reading," p. 23; M. Beran, *Amer. J. Phys.*, 35, p. 242, 1967.

The Liouville operator has been defined in such a way as to bring the Liouville equation into the form of the Schrödinger equation. A formal, and sometimes useful, solution to Eq. (18–7) is

$$f_N(\mathbf{p}, \mathbf{r}, t) = e^{-iLt} f_N(\mathbf{p}, \mathbf{r}, 0) \tag{18–9}$$

Note that the operator $\exp(-iLt)$ displaces f_N ahead a distance t in time. This operator is called the *time displacement operator* of the system.

18–2 REDUCED DISTRIBUTION FUNCTIONS

Once we have the distribution function $f_N(p, q, t)$, we may compute the ensemble average of any dynamical variable, $A(p, q, t)$, from the equation

$$\langle A(t) \rangle = \int A(p, q, t) f_N(p, q, t)\, dp\, dq \tag{18–10}$$

It turns out that the dynamical variables of interest are functions of either the coordinates and momenta of just a few particles or can be written as a sum over such functions. A familiar example of this is the total intermolecular potential of the system. To a good approximation, this can be written as a sum over pair-wise potentials, and so

$$\langle U \rangle = \sum_{i,j} \int \cdots \int u(\mathbf{r}_i, \mathbf{r}_j) f_N(\mathbf{r}_1, \ldots, \mathbf{p}_N, t)\, d\mathbf{r}_1 \cdots d\mathbf{p}_N \tag{18–11}$$

We encountered similar integrands when we studied the equilibrium theory of liquids. There we integrated over the coordinates of all the particles except i and j and called the resulting function of $\mathbf{r}_i$ and $\mathbf{r}_j$ a radial distribution function. We do the same thing here. We define *reduced distribution functions* $f_N^{(n)}(\mathbf{r}_1, \ldots, \mathbf{r}_n, \mathbf{p}_1, \ldots, \mathbf{p}_n, t)$ by

$$f_N^{(n)}(\mathbf{r}_1, \ldots, \mathbf{r}_n, \mathbf{p}_1, \ldots, \mathbf{p}_n, t) = \frac{N!}{(N-n)!} \int \cdots \int f_N(\mathbf{r}_1, \ldots, \mathbf{p}_N, t)\, d\mathbf{r}_{n+1} \cdots d\mathbf{r}_N\, \mathbf{p}_{n+1} \cdots d\mathbf{p}_N \tag{18–12}$$

We shall usually drop the N subscript and furthermore write this simply as $f^{(n)}(\mathbf{r}^n, \mathbf{p}^n, t)$. Usually only $f^{(1)}$ and $f^{(2)}$ are necessary, and therefore we want to derive an equation for $f^{(1)}$ and $f^{(2)}$. To do this, write the force $\mathbf{F}_j$ appearing in the Liouville equation as the sum of the forces due to the other molecules in the system $\sum_i \mathbf{F}_{ij}$ and an external force $\mathbf{X}_j$. Then multiply through by $N!/(N-n)!$ and integrate over $d\mathbf{r}_{n+1} \cdots d\mathbf{r}_N\, d\mathbf{p}_{n+1} \cdots d\mathbf{p}_N$ to get (Problem 18–2)

$$\frac{\partial f^{(n)}}{\partial t} + \sum_{j=1}^{n} \frac{\mathbf{p}_j}{m_j} \cdot \nabla_{\mathbf{r}_j} f^{(n)} + \sum_{j=1}^{n} \mathbf{X}_j \cdot \nabla_{\mathbf{p}_j} f^{(n)} + \frac{N!}{(N-n)!} \sum_{i,j=1}^{N} \int \cdots \int \mathbf{F}_{ij} \cdot \nabla_{\mathbf{p}_j} f\, d\mathbf{r}_{n+1} \cdots d\mathbf{r}_N\, d\mathbf{p}_{n+1} \cdots d\mathbf{p}_N = 0 \tag{18–13}$$

We have used the fact that f vanishes outside the walls of the container and when $\mathbf{p}_i = \pm\infty$. The last term in Eq. (18–13) can be broken up into two parts:

$$\sum_{i,j=1}^{n} \mathbf{F}_{ij} \cdot \nabla_{\mathbf{p}_j} f^{(n)} + \frac{N!}{(N-n)!} \sum_{j=1}^{n} \sum_{i=n+1}^{N} \int \cdots \int \mathbf{F}_{ij} \cdot \nabla_{\mathbf{p}_j} f\, d\mathbf{r}_{n+1} \cdots d\mathbf{r}_N\, d\mathbf{p}_{n+1} \cdots d\mathbf{p}_N$$

The second term here can be written as

$$\sum_{j=1}^{n} \iint \mathbf{F}_{j,n+1} \cdot \nabla_{\mathbf{p}_j} f^{(n+1)} \, d\mathbf{r}_{n+1} \, d\mathbf{p}_{n+1}$$

Putting all this together finally gives an exact equation for $f^{(n)}$, namely, (Problem 18–3),

$$\frac{\partial f^{(n)}}{\partial t} + \sum_{j=1}^{n} \frac{\mathbf{p}_j}{m_j} \cdot \nabla_{\mathbf{r}_j} f^{(n)} + \sum_{j=1}^{n} \mathbf{X}_j \cdot \nabla_{\mathbf{p}_j} f^{(n)} + \sum_{i,j=1}^{n} \mathbf{F}_{ij} \cdot \nabla_{\mathbf{p}_j} f^{(n)} + \sum_{j=1}^{n} \iint \mathbf{F}_{j,n+1} \cdot \nabla_{\mathbf{p}_j} f^{(n+1)} \, d\mathbf{r}_{n+1} \, d\mathbf{p}_{n+1} = 0 \tag{18–14}$$

This is the so-called Bogoliubov, Born, Green, Kirkwood, Yvon (BBGKY) hierarchy. This is the time-dependent generalization of the hierarchy that we derived earlier in the equilibrium theory of fluids. In fact, if one assumes that

$$f^{(n)} = g^{(n)}(\mathbf{r}_1, \ldots, \mathbf{r}_n) \exp\left\{-\frac{1}{2mkT} \sum_{j=1}^{n} p_j^{\,2}\right\}$$

multiplies Eq. (18–14) through by $\mathbf{p}_i$, $1 \le i \le n$, and integrates over all momenta, one obtains the equilibrium hierarchy for $g^{(n)}(\mathbf{r}_1, \ldots, r_n)$ (Problem 18–4). It would seem natural at this point to truncate this hierarchy by some sort of a superposition approximation, but so far this approach has not been successful.* We shall end up deriving approximate equations for $f^{(1)}$ and $f^{(2)}$.

Everything we have done up to now has been independent of density; i.e., it has been applicable to any density. Now we shall specialize to systems of dilute gases.

18–3 FLUXES IN DILUTE GASES

In a dilute gas, most of the molecules are not interacting with any other molecule and are just traveling along between collisions. Because of this, the macroscopic properties of a gas depend upon only the singlet distribution function $f_j^{(1)}(\mathbf{r}, \mathbf{p}_j, t)$. The subscript j here denotes the singlet distribution function of species j. This is the central distribution function of any theory of transport in dilute gases. In this section we shall define a number of averages over $f_j^{(1)}$ and derive molecular expressions for the important flux quantities in terms of integrals over $f_j^{(1)}$. Since we shall be concerned only with gases in this and the following sections, we shall drop the superscript (1) from here on. We shall also write our equations in velocity space rather than momentum space, and so the distribution function of interest becomes $f_j(\mathbf{r}, \mathbf{v}_j, t)$. We shall renormalize f_j such that the integral of this distribution function over all velocities is the number density of j particles at the point $\mathbf{r}$ at time t, i.e.,

$$\rho_j(\mathbf{r}, t) = \int f_j(\mathbf{r}, \mathbf{v}_j, t) \, d\mathbf{v}_j \tag{18–15}$$

Furthermore, if N_j is the total number of j molecules in our system, then

$$N_j = \iint f_j(\mathbf{r}, \mathbf{v}_j, t) \, d\mathbf{r} \, d\mathbf{v}_j \tag{18–16}$$

We shall now define a number of important average velocities. $\mathbf{v}_j$ is the *linear*

* See, for example, R. G. Mortimer, *J. Chem. Phys.*, **48**, p. 1023, 1968.

velocity of a molecule of species j; i.e., it is the velocity with respect to a coordinate system fixed in space. The average velocity is given by

$$\mathbf{v}_j(\mathbf{r}, t) = \frac{1}{\rho_j} \int \mathbf{v}_j f(\mathbf{r}, \mathbf{v}_j, t)\, d\mathbf{v}_j \tag{18–17}$$

and represents the macroscopic flow of species j. The *mass average velocity* is defined by

$$\mathbf{v}_0(\mathbf{r}, t) = \frac{\sum_j m_j \rho_j \mathbf{v}_j}{\sum_j m_j \rho_j} \tag{18–18}$$

Note that the denominator here is the mass density $\rho_m(\mathbf{r}, t)$. This velocity is often called the flow velocity or stream velocity. The momentum density of the gas is the same as if all the molecules were moving with velocity $\mathbf{v}_0$. The *peculiar velocity* is the velocity of a molecule relative to the flow velocity. The peculiar velocity $\mathbf{V}_j$ is

$$\mathbf{V}_j = \mathbf{v}_j - \mathbf{v}_0 \tag{18–19}$$

The average of this peculiar velocity is the *diffusion velocity* (Problem 18–5). Clearly,

$$\overline{\mathbf{V}}_j = \frac{1}{\rho_j} \int (\mathbf{v}_j - \mathbf{v}_0) f_j(\mathbf{r}, \mathbf{v}_j, t)\, d\mathbf{v}_j \tag{18–20}$$

It is easy to show that (Problem 18–6)

$$\sum_j \rho_j m_j \overline{\mathbf{V}}_j = 0 \tag{18–21}$$

When we studied the elementary kinetic theory of gases, we saw that the various transport coefficients were related to molecular transport of mass, momentum, and kinetic energy. Let these molecular properties be designated collectively by ψ_j, where j refers to the particular species. We now derive expressions for the fluxes of these properties. Figure 18–1 shows a surface dS moving with velocity $\mathbf{v}_0$. The quantity $\mathbf{n}$ is a unit vector normal to dS, and $d\mathbf{S} = \mathbf{n}\, dS$. All the molecules that have velocity $\mathbf{V}_j = \mathbf{v}_j - \mathbf{v}_0$ and that cross dS in the time interval $(t, t + dt)$ must have been in a cylinder of length $|\mathbf{V}_j|\, dt$ and base dS. This cylinder is shown in Fig. 18–1 and has a volume $(\mathbf{n} \cdot \mathbf{V}_j)\, dS\, dt$. Since there are $f_j\, d\mathbf{v}_j$ molecules per unit volume with relative velocity $\mathbf{V}_j$, the number of j molecules that cross dS in dt is given by

$$(f_j\, d\mathbf{v}_j)(\mathbf{n} \cdot \mathbf{V}_j)\, dS\, dt$$

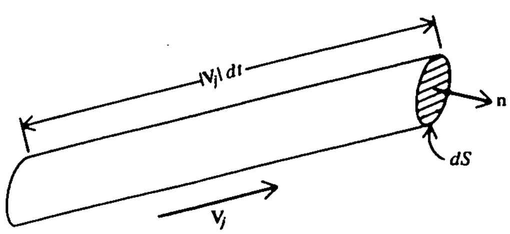

Figure 18–1. **The cylinder containing all those molecules of species j with velocity $\mathbf{V}_j$, which cross the surface dS during the time interval dt.** (From J. O. Hirschfelder, C. F. Curtiss, and R. B. Bird, *Molecular Theory of Gases and Liquids*. New York: Wiley, 1954.)

If each molecule carries with it a property ψ_j, then the flux of this property is

$$\psi_j f_j(\mathbf{n} \cdot \mathbf{V}_j)\, d\mathbf{v}_j$$

and the total flux across this surface is

$$\text{total flux} = \int \psi_j f_j(\mathbf{n} \cdot \mathbf{V}_j)\, d\mathbf{v}_j = \mathbf{n} \cdot \int \psi_j f_j \mathbf{V}_j\, d\mathbf{v}_j = \mathbf{n} \cdot \boldsymbol{\psi}_j \tag{18-22}$$

The vector $\boldsymbol{\psi}_j$,

$$\boldsymbol{\psi}_j = \int \psi_j f_j \mathbf{V}_j\, d\mathbf{v}_j \tag{18-23}$$

is called the flux vector associated with the property ψ_j. The component of this vector in any direction is the transport of the property ψ_j in that direction. Let us now consider the various examples of ψ_j.

TRANSPORT OF MASS

In this case, $\psi_j = m_j$, and

$$\boldsymbol{\psi}_j = m_j \int f_j \mathbf{V}_j\, d\mathbf{v}_j = \rho_j m_j \overline{\mathbf{V}_j} \equiv \mathbf{j}_j \tag{18-24}$$

TRANSPORT OF MOMENTUM

Here $\psi_j = m_j V_{jx}$, and

$$\boldsymbol{\psi}_j = m_j \int V_{jx} f_j \mathbf{V}_j\, d\mathbf{v}_j = \rho_j m_j \overline{V_{jx} \mathbf{V}_j} \tag{18-25}$$

which is the flux of the x-component of momentum relative to $\mathbf{v}_0$. The flux of momentum is a pressure, which has components

$$(p_j)_{xx} = \rho_j m_j \overline{V_{jx} V_{jx}}$$

$$(p_j)_{xy} = \rho_j m_j \overline{V_{jx} V_{jy}}, \text{ etc}$$

or, in general,

$$\mathsf{p}_j = \rho_j m_j \overline{\mathbf{V}_j \mathbf{V}_j} \tag{18-26}$$

which is the partial pressure tensor of the jth species.

TRANSPORT OF KINETIC ENERGY

$$\psi_j = \tfrac{1}{2} m_j V_j^2$$

and

$$\boldsymbol{\psi}_j = \frac{m_j}{2} \int v_j^2 \mathbf{V}_j f_j\, d\mathbf{v}_j = \tfrac{1}{2} \rho_j m_j \overline{V_j^2 \mathbf{V}_j} = \mathbf{q}_j \tag{18-27}$$

the heat flux vector of the jth species.

It should be clear at this point that once we have an expression for $f_j(\mathbf{r}, \mathbf{v}_j, t)$, we can calculate all the fluxes and hence all the transport properties of a dilute gas. What we need now is f_j, or at least an equation that gives f_j as its solution. The only equation we have up to now is Eq. (18-14) with $n = 1$, and it can be seen that this also contains $f_j^{(2)}$. As we said earlier, nobody has found a successful way to uncouple this system. In the next section we shall derive an equation for f_j, the Boltzmann equation, which is the fundamental equation of the rigorous kinetic theory of gases.